Name:

# Fall 2017 Math 245 Exam 3

Please read the following directions:
Please write legibly, with plenty of white space. Please print your name on the designated line, similarly to your quizzes (last name(s) in ALL CAPS). Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 1:00 and will last at most 50 minutes; pace yourself accordingly. Please leave **only** at one of the designated times: 1:20pm, 1:40pm, or 1:50pm. At all other times please stay in your seat (emergencies excepted), to ensure a quiet test environment for others. Good luck!

| Problem | Min Score | Your Score | Max Score |
|---|---|---|---|
| 1. | 5 | | 10 |
| 2. | 5 | | 10 |
| 3. | 5 | | 10 |
| 4. | 5 | | 10 |
| 5. | 5 | | 10 |
| 6. | 5 | | 10 |
| 7. | 5 | | 10 |
| 8. | 5 | | 10 |
| 9. | 5 | | 10 |
| 10. | 5 | | 10 |
| Exam Total: | 50 | | 100 |
| Quiz Ave: | 50 | | 100 |
| Overall: | 50 | | 100 |

Note: 44% of the course grade comes after Exam 3.

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. recurrence

b. $\Omega$

c. $\Delta$

d. $=$ (for sets)

Problem 2. Carefully define the following terms:

a. disjoint

b. equicardinal

c. Distributivity Theorem (for sets)

d. De Morgan's Law Theorem (for sets)

Problem 3. Let $S = \{a, b\}$. Give a two-element subset of $2^{2^S}$.

Problem 4. Suppose that a recurrence satisfies the relation $T_n = 4T_{n/2} + n^2$. Determine what, if anything, the Master Theorem tells us.

Problem 5. Let $R, S, T$ be sets, with $S \subseteq T$. Prove that $R \cap S \subseteq R \cap T$.

Problem 6. Let $R, S, U$ be sets, with $R \subseteq S \subseteq U$. Prove that $S^c \subseteq R^c$.

Problem 7. Prove or disprove: For all sets $R, S$, $R \times S = S \times R$.

Problem 8. Solve the recurrence given by $a_0 = a_1 = 1, a_n = 5a_{n-1} - 6a_{n-2}$ $(n \geq 2)$.

Problem 9. Let $a_n = 3n^2 + 7$. Prove that $a_n = \Theta(n^2)$.

Problem 10. Let $R, S, T$ be sets. Prove that $R \times (S \cup T) \subseteq (R \times S) \cup (R \times T)$.
Note: Do not use Theorem 9.13, which states that $R \times (S \cup T) = (R \times S) \cup (R \times T)$.